Having attained the general failure function, the tangent modulus to the failure surface at $\theta = 0$ (non-pure shear state) at various confining pressures may be determined. This in effect provides a correlation between the friction angle and the mean pressure of the following form:

$$\phi = 26.3(0.61)^{I_1/f_c} + 31.2 \quad \text{for} \quad 0 \le \frac{I_1}{f_c} \le 4 \qquad (11)$$

In other words, the friction angle is approximately equal to 58° for unconfined state, and under confining pressures of about 4 times the compressive strength it reduces to about 35°.

4 CONCLUSION

Although the use of a two parameter failure model such as Mohr-Coulomb in its original form for RCC, may be convenient, but it is far too simplistic for appropriate and optimized design procedure. It is therefore justified to obtain a better appreciation of the failure behaviour of such materials through more comprehensive laboratory and theoretical studies.

Four parameter models provide a better approximation of the behaviour of RCC. However, in order to circumvent the complexities of such models in the routine design procedures, the essence of this model is used in evaluation of the friction angle.

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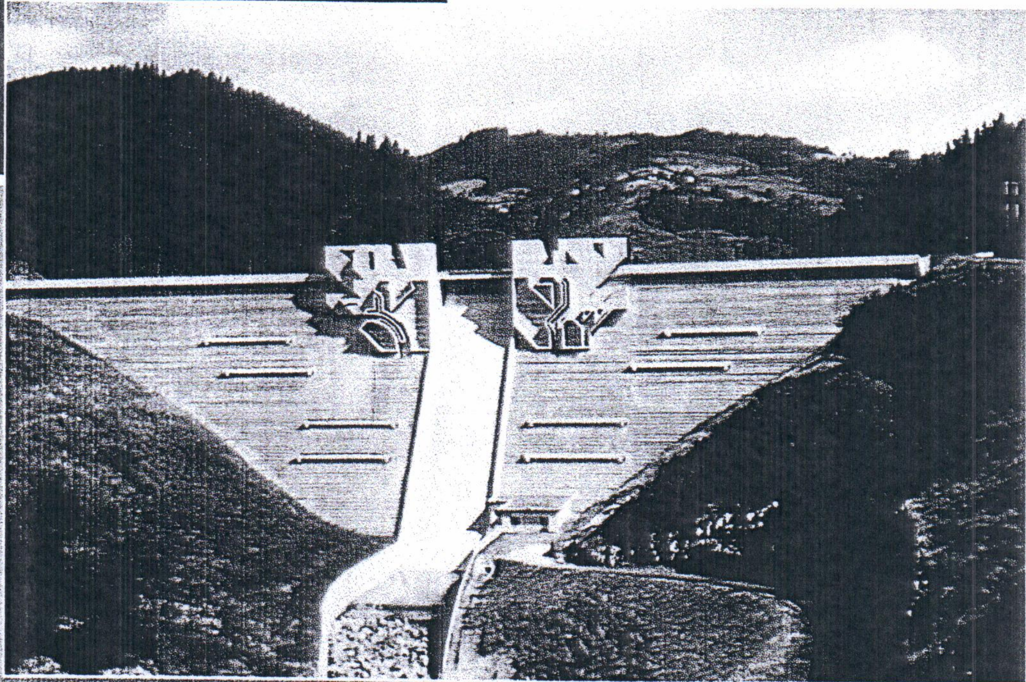

Roller Compacted Concrete Dams

Edited by:
L. Berga, J.M. Buil,
C. Jofré & S. Chonggang

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Constitutive modelling of Roller Compacted Concrete

F. Kalantary & P. Sadeghi
Moshanir Power Eng. Consultant, Iran

ABSTRACT: Although the use of simple failure criterion such as Mohr-Coulomb, in the numerical analysis and design of Roller Compacted Concrete (RCC) dams may be instructive, but by no means suffices. Therefore it seems right to seek a constitutive model more capable of capturing the true behavioral pattern of RCC material. On the other hand such comprehensive models have proven too tedious to be accepted readily by the industry. It is thus proposed here to employ the four parameter Hsieh-Ting-Chen (1982) criterion, calibrated with the published experimental data on RCC by Peng et al. (1997). Having attained the basic criterion, the Mohr-Coulomb parameters are evaluated at various mean pressures for a constant value of Lode's angle (θ), which corresponds with the stress path of tri-axial compression test. Thereby an exponential relationship is evaluated between the angle of internal friction and mean pressure, normalized with uni-axial compressive strength (f_c) of the material.

1 INTRODUCTION

Numerical techniques have found a permanent position among engineering calculations, and are currently being used extensively in dam engineering designs. However, these numerical calculations are as accurate as the constitutive model that describes the material behavior in them.

Constitutive models for Roller Compacted Concrete (RCC), as a relatively new type of material in dam construction has yet to be defined and verified, if any conclusive results are to be obtained from numerical analysis of such dams.

The first step in constitutive modelling of a material, beside its elastic behaviour, is to derive the yield function, describing the onset of plastic deformation and crack propagation.

Experimental evidence show that the mechanical behavior of Roller Compacted Concrete resembles concrete rather than soil [Peng et al. (1997)].

Soil is a frictional material that gains shear strength with increase of confining pressure, and although this variation is non-linear, Mohr-Coulomb criteria is still being used with minor empirical modification of parameters in earth dam design.

However, the shear strength of RCC, or for that matter concrete, is not derived solely from frictional characteristics of the material, rather the cementation and the bond formed between the cement-flyash/pozzolana gel and the aggregate is the major contributing factor in shear strength.

Furthermore, there is a major difference between the hardening law of concrete and soil; plastic volumetric strains are the main contributing factor to hardening of granular materials, whereas any significant deformation, be it volumetric, after the initial set of concrete will cause reduction of strength, i.e. softening. This is the reason many of the models neglect hardening issues and assume a rigid-plastic behavior for concrete.

Thus it seems rudimentary to use purely frictional yield criteria such as Mohr-Coulomb for RCC dams stress analysis, specially under dynamic loading.

In recent decades a number of yield functions have been proposed for concrete and there now exist well-developed plasticity theory for concrete, a short review of which will now be presented.

2 CONCRETE YIELD CRITERION

Concrete is composite material and its yield function is affected by many characteristic properties of its constituents. However considering all possible influencing factors is not only a formidable task but also is undesirable in view of complications that this will cause in the use of the model. Instead, representative parameters such as compressive strength are very suitable in the definition of the yield function.

Yield functions for concrete have been determined through experimental as well as theoretical studies and it is now commonly believed that concrete yield function is of the following general form:

$$f(I_1, J_2, J_3) = 0 \qquad (1)$$

where $I_1 = \sigma_{ii}$, $J_2 = \frac{1}{2} s_{ij} s_{ij}$, $J_3 = \frac{1}{3} s_{ij} . s_{jk} . s_{ki}$

and σ_{ii}, s_{ij} are the principal stresses and the deviatoric stresses respectively.

Based on two dimensional experimental results by Kupfer et al. (1969), and Tasuji et al. (1978), and three dimensional experimental results by Mills et al. (1970) and Looney and Gratsle [Chen, 1994] more specific definitions of yield surface for concrete has been outlined. Among these one may refer to Bresler-Prister, and Willam-Warnke three parameter models and Ottosen (1977) and Hsieh-Ting-Chen (1982) four parameter models.

The parameters used in these models may be determined using the results of uni-axial, bi-axial or tri-axial compressive and shear tests.

A more detailed description of the above-mentioned models may be found in Chen (1994).

Comparison of the models has proven that in general the four-parameter models are more versatile and give a better approximation.

The Ottosen (1977) four-parameter function may be state in a non-dimensional form as:

$$f(I_1, J_2, \theta) = a\frac{J_2}{f_c^2} + \lambda\frac{\sqrt{J_2}}{f_c} + b\frac{I_1}{f_c} - 1 = 0 \qquad (2)$$

where

$$\lambda = \begin{cases} k_1.Cos\left[\frac{1}{3}.Cos^{-1}(k_2.Cos3\theta)\right] & Cos3\theta \geq 0 \\ k_1.Cos\left[\frac{\pi}{3} - \frac{1}{3}Cos^{-1}(-k_2.Cos3\theta)\right] & Cos3\theta \leq 0 \end{cases}$$

$$Cos3\theta = \frac{3\sqrt{3}}{2} . \frac{J_3}{J_2^{3/2}}$$

f_c is the ultimate uni-axial compressive strength and the coefficients a, b, k_1 and k_2 are the main parameters which have been determined for ordinary concrete according to the experimental results provided by Kupfer et al. (1969), Richart et al. and Balmer (Ottosen 1977) for $f_t/f_c = 0.1$: a = 1.2759, b = 3.1962, k_1 = 11.7365, k_2 = 0.9801.

Although this yield function has had a good correlation with experimental data, its use in a general elasto-plastic algorithm has been restrictive due to its cumbersome form: λ is defined in terms of θ and two other parameters.

For this reason Hsieh et al. (1982) proposed a variation on this function by substituting λ with a function in terms of θ. The failure function in the Haigh-Westergarrd coordinates is stated in the following form:

$$f(\rho, r, \theta) = ar^2 + (\alpha.Cos\ \theta + \beta)r + b\rho - 1 \qquad (3)$$

where $\rho = \frac{I_1}{3}$ and $r = \sqrt{2J_2}$

Figure 1 shows a general view of this yield surface in the three dimensional principal stress space and the schematic meridian representations of yield surface in (ρ–r) space with constant Lode's angles are shown in Figure 2.

In effect the coefficient of the second term has been equated to (α Cos θ + β = k). With this definition the cross-section of the yield surface in the deviatoric plane (π-plane) may change from a circle (i.e. α = 0, β = 1 and thus k = r) to triangle (i.e. α = 1, β = 0 and thus k = r · Cosθ), as shown in Figure 3.

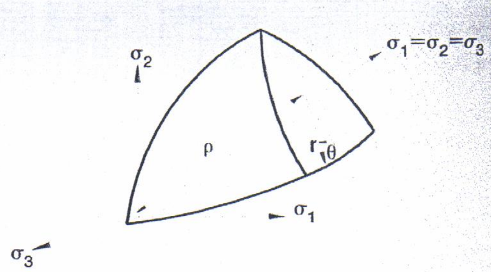

Figure 1. Schematic representation of the yield surface.

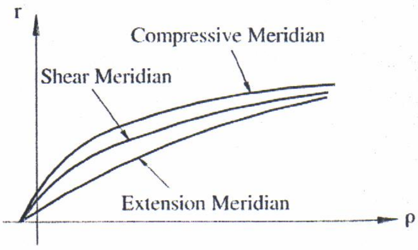

Figure 2. Failure envelope in (ρ–r) space..

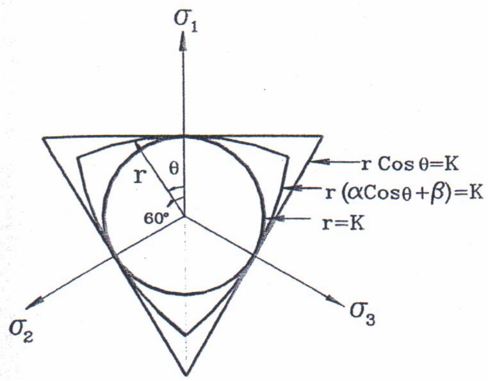

Figure 3. π-plane view of failure surface.

An advantage of this definition is that the yield surface may be stated in a non-dimensional form, in terms of the invariants of stresses and compressive strength of concrete:

$$A\frac{J_2}{(f_C)^2}+B\frac{\sqrt{J_2}}{f_C}+C\frac{\sigma_1}{f_C}+D\frac{I_1}{f_C}-1=0 \quad (4)$$

In the above equation A, B, C and D are the model parameters and σ_1 is the maximum principal stress with tensile stresses being positive.

The parameters have been determined for ordinary concrete on the basis of the experimental results of Kupfer et al. (1969) and Mills et al. (1970). [Hsieh et al. (1982)]

Uni-axial compression test;

$$\sigma_1=-f_C, \quad \sigma_2=\sigma_3=0 \quad (5.1)$$

Uni-axial tensile test;

$$\sigma_1=0.1f_c, \quad \sigma_2=\sigma_3=0 \quad (5.2)$$

Bi-axial compression test;

$$\sigma_1=\sigma_2=-1.15f_c, \quad \sigma_3=0 \quad (5.3)$$

Tri-axial compression test;

$$\sigma_1=-4.2f_c \quad \sigma_2=\sigma_3=-0.8f_c \quad (5.4)$$

therefore:
A = 2.011, B = 0.971, C = 9.141, D = 0.231.

3 CALIBRATION OF THE MODEL FOR RCC

In order to calibrate the model for RCC, appropriate experimental results must be considered. Published experimental data as well as empirical relationships are used here to derive the model parameters.

One of the most comprehensive sets of experimental work on failure behaviour of RCC was carried out by Peng et al. (1997). In this work the results of uni-axial tension/compression, direct shear and tension/compression-shear tests on common RCC as well as jointed RCC at ages of 91 and 180 days were presented.

Here, use is made of the results of common RCC at the age of 91-days. Mean uni-axial tensile, compressive and direct shear strength of the specimens were found to be 2.34 MPa, 22.4 MPa and 3.34 MPa respectively.

However the choice of data sets to be employed requires an insight. For example, should only the data corresponding to bi-axial tests (or for that matter any specific single test condition) be used, the simultaneous equations will become singular.

Based on the following specific stress paths, the model parameters are found:

Uni-axial compressive strength tests provide one equation in the set of simultaneous equations:

$$\sigma_1=-f_C, \quad \sigma_2=\sigma_3=0 \quad (6.1)$$

The stress path to failure in the uni-axial tensile test provides another point on the failure surface. However since direct tensile tests on brittle materials such as concrete are non-standard, indirect test (Brazilian test) is commonly used to evaluate tensile strength. In any case the results have a relatively wide scatter. Nevertheless, Hansen (1991) has showed that the ratio of tensile strength to unconfined compressive strength (f_t/f_c) is approximately equal to 10% for ordinary concrete and the results provided by Peng et al.(1997) for common RCC and jointed RCC show that this ratio ranges between 6–11%, the lower values being associated with jointed RCC and higher ages. Furthermore, it has been found that for brittle materials this ratio is inversely proportional to the strength (Dunstan 1999, Kalantary et al. 2002). That is for lean concrete the (σ_1/f_c) ratio has a higher value than ordinary concrete. Therefore it is proposed here to assume a ratio of 0.105. This is in accordance with the results provided by Peng et al. (1997) on common RCC at an age of 91-days. Therefore:

$$\sigma_1=0.105f_c, \quad \sigma_2=\sigma_3=0 \quad (6.2)$$

The results of bi-axial compressive tests yield yet another equation for the solution of the simultaneous

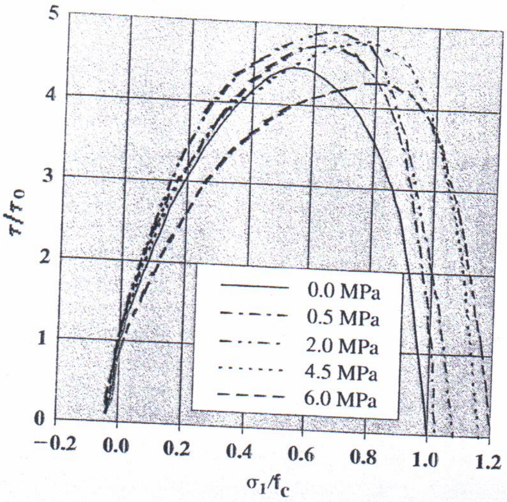

Figure 4. RCC failure envelope (Peng et al. 1997).

equations. Failure envelopes reproduced in Figure 4 from the results of Peng et al. (1997), can help to provide another set of data for calibration of the model parameters. In this Figure (σ_1/f_c) is the ratio of principal maximum normal stress to the unconfined compressive strength and (τ/τ_o) is the ratio of general shear strength to the unconfined direct shear strength.

From the failure envelope for $\sigma_2 = 6$ MPa, at zero applied shear stress, the ratio of (σ_1/f_c) is found to be 1.2. (i.e. the compressive strength is increased by 20% from the uni-axial compressive strength). Thus:

$$\sigma_1 = -1.2 f_c,\ \sigma_2 = -0.27 f_c,\ \sigma_3 = 0 \tag{6.3}$$

Other bi-axial test results may also be used instead. These give the same result since the rate of variation of σ_1/σ_2 is constant (Figure 5).

The results of tri-axial compression tests should complete the set of equations for evaluation of the model parameters. However, due to the lack tri-axial test results on RCC samples the stress path for ordinary concrete is adopted here. (i.e. equation 5.4).

By using the above results in the set of simultaneous equations the parameters A, B, C and D may be evaluated for common RCC: A = 1.929, B = 1.327, C = 8.327, D = 0.410, and the failure function may be stated as:

$$\frac{1.929 J_2}{(f_c)^2} + \frac{1.327\sqrt{J_2}}{f_c} + \frac{8.327\sigma_1}{f_c} + \frac{0.410 I_1}{f_c} - 1 = 0 \tag{7}$$

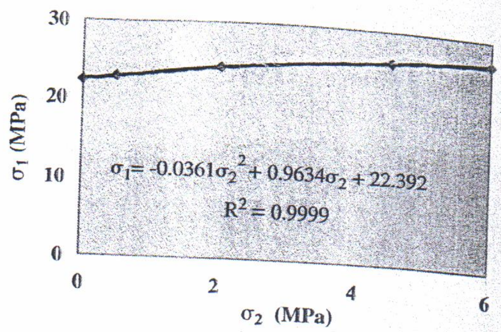

Figure 5. Failure envelope in σ_1–σ_2 space from bi-axial test results (Peng et al. 1997).

The evaluated values are close to that of ordinary concrete provided by Hsieh et al. (1982).

Now by substituting σ_1 with the corresponding stress invariants:

$$\sigma_1 = 2\sqrt{\frac{J_2}{3}}.Sin\left(\theta + \frac{2\pi}{3}\right) + \frac{I_1}{3} \tag{8}$$

The failure function may be stated in terms of I_1, J_2 and θ:

$$\frac{1.929 J_2}{f_c} + \sqrt{J_2}\left[1.327 - 4.808.Sin\theta + 8.327 Cos\theta\right] + 3.186 I_1 = f_c \tag{9}$$

and the coefficients of the associated flow rule (Owen & Hinton 1982) may be stated as:

$$C_1 = 2.818 \tag{10.1}$$

$$C_2 = \frac{3.209\sqrt{J_2}}{f_c} + 1.376 + 7.468 \left[Cos\theta - \frac{Sin\theta}{\sqrt{3}} + Tan3\theta\left(\frac{Cos\theta}{\sqrt{3}} + Sin\theta\right)\right] \tag{10.2}$$

$$C_3 = \frac{3.734\left(Cos\theta + \sqrt{3}\right)}{\sqrt{J_2}\,Cos3\theta} \tag{10.3}$$

The model may now readily be incorporated into a finite element programme.